An Approach to Blend Surfaces

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Abstract

In this paper, we present an application of a space mapping technique for surface reconstruction (more precisely: reconstruction of missing parts of a real geometric object represented by volume data). Using a space mapping technique, the surface of a given model, in particular tooth shape is fitted by a shape transformation to extrapolate the remaining surface of a patient's tooth with occurring damage such as a "drill hole." The genetic algorithm minimizes the error of the approximation by optimizing a set of control points that determine the coefficients for spline functions, which in turn define a space transformation. The fitness function to be minimized consists of two components. First one is the error between the blended surface of an object and the surface of the object to be blended in some predefined points. The second is a component that is responsible for the bending energy being minimized.

Keywords: Constructive solid geometry, volume modeling, space mapping, surface reconstruction, computer-aided restoration design

1. Introduction

This paper extends the work of Savchenko and Shmitt [1] where an application of 3D-space mapping technique and numerical optimization with a specially designed genetic algorithm to a problem concerning computer aided design (CAD) in dentistry was presented. The applications considered and approaches proposed in this paper and in the article [1] are sufficiently-different. In [1] the task was to find a functionally transformed occlusal surface (an inlay part) of a model tooth that best matches the remaining occlusal surface of a treated tooth. In this project, we attempt to approach the well-known problem of "fulfillment" of missing data. The surface reconstruction problem can be expressed as follows:

Given shapes S_1, S_2, ... , find a blending surface S_b that is a reasonable approximation of the surface satisfying a set of constrains.

Important examples of fitting surface data have been intensively investigated in the past few years. An overview can be found in the paper [2] by Hermann et al. Various strategies were proposed to minimize user interaction; however, it is noticed in [2] that surface fitting of complex free-form shapes still remains a largely unsolved problem of great practical importance.

Traditionally, constructive solid geometry (CSG) modeling uses simple geometric objects for a base model, which can be further manipulated by implementing a certain collection of operations such as set-theoretic operations, blending, or offsetting. Recently, a great attention was paid to new implicit surface creation methods based on the use of radial based functions (RBF) for interpolating scattered data. Like other implicit surfaces they can be used for CSG modeling. Whatever complex operations have been applied to a geometric object, which can be given by a voxel raster, by elevation data, or by interpolated data, an equivalent volume data can be generated. We use the term *volume model* to refer to the wide class of surfaces that like other implicit surface descriptions can be used for CSG [3]. Objects defined in this manner are not regularized solids, as is required in traditional constructive solid geometry modeling, but have a function description.

In this paper we propose that RBFs offer a mechanism to get extrapolated points of a blended surface for various parts of a reconstructed object that can be used as “CSG components” to design a volume model.

Here let us explain the motivation behind this project. In spite of a flurry of activity in the field of scattered data reconstruction and interpolation, this matter remains a difficult and computationally expensive problem. Many recent works have focused on simplifying large data sets and researchers usually state that RBF methods guarantee automatic mesh repair or interpolation of large irregular holes. Our experiments show that reconstruction of a hemisphere by using one pole point and points belonging two parallel cross sections gives a maximum error of about 20 percent. A vast amount of literature is devoted to the subject of scattered data interpolation. Different interpolations can generate very different output for the same data set. For example, grids interpolated by Shepard’s method [4] miss some details of the original data. In most applications a Delaunay triangulation is used for object reconstruction. Unfortunately, this method has some serious drawbacks resulting in a confusing image because sometimes an object’s features can be stretched and distorted. Even with the elimination of large triangles, the reconstructed shape remains convex-looking, as noted in [5].

Another approach to surface reconstruction is skeletal. An implicit surface generated by point skeletons may be fit to a set of surface points [6], but this method is rather time consuming.

One other approach is to use methods of scattered data interpolation, based on the minimum-energy properties [7], [8], [9], [10], [11], [12]. The benefits of modeling 3D surfaces with the help of radial basis functions (RBF) have been recognized in [13]. They were adapted for computer animation [14], and medical applications [15], [16] and were first applied to reconstruct implicit surfaces by Savchenko et al.

[17]. However, the required computational work is proportional to the number of grid nodes and the number of scattered data points. Special methods to reduce the processing time were developed for thin plate splines and discussed in [18], [19], see also recent publications [20], [21], [22].

Terracing and ringing problems are well discussed in the literature concerning terrain reconstruction. Partial differential equations (PDE) are widely used to model surfaces subject to certain constrains, see [23], [24]. A well-founded mathematically set-level approach has found applications in shape design and data fitting. Whitaker and Breen [25] give a highly exhaustive overview of related to this matter works; their paper presents an improved numerical algorithm and solves particular problems in geometric modeling. In particular, level set models with a positive-curvature flow are used to create a smooth blend (fillet) between solid objects. Gousie and Franklin give an extended overview of PDE methods and present two new methods for approximating elevation data from contours data to a grid [26]. They tested various interpolation/approximation methods designed with the following properties: the ability to handle realistic amounts of data and to minimize artifacts (terracing and ringing). The intermediate contours method produces a new contour between existing successive contours and a thin-plate approximation is applied to complete the surface between the new contours. In the gradient lines method, gradient lines are calculated from the surface produced by the thin-plate interpolation. At the next step of interpolating elevations along the whole gradient, the RBFs or thin-plate approximation is applied once again. The proposed methods appear visually smoother, with undesirable terracing effects much reduced.

Among a wide array of various techniques and solutions, the most famous one is the snake model proposed by Kass, Witkin and Terzopoulos [27]. We refer the interested reader to the paper by Duan and Qin [28]. A technique presented in [28] is capable of recovering geometric shape of unknown topology either from volumetric data or from range data. However, it cannot overcome the problem of missing data; through the minimization process of the cost function, their model will inflate like a balloon until it reaches the boundary of the modeled objects.

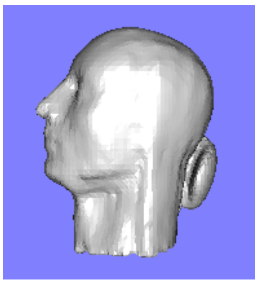


Figure 1. Illustration of an improper reconstruction by compactly supported RBFs

Recently special methods, so called compactly supported RBFs, aiming to reduce processing time were developed. For more references see [29]. However, if we have

a gap in the data set, we face the problem of evaluating the functions at extra points; if the radius of support is quite large, then the cycle associated with the matrix of linear equations will include nearly all the points from the input data. For more references see [30]. Figure 1 illustrates an artifact (can be observed on the right-hand side of the reconstructed area) stipulated an inappropriate choice of the radius of support. Let us note that we have a lack of data in lower part of the "head." Thus, the work required for correct reconstruction of the object is becoming nearly proportional to number of all scattered data points. The amount of computations becomes significant, even for moderate number of nodes.
The goal in occlusal surface modeling for restoration is to reproduce a "reasonable" reconstructed shape surrounding a missing part of the restoration area. In [1], an example of applying a monotone formula [31] demonstrates how one can use an exact analytic description (a polygon) to reproduce inlay parts of treated teeth. A variety of techniques can be applied to let the user create volume models applying geometric operations. In this work, for simplicity we are using a "drill hole."

In what follows, we shall review a class of techniques that can be used in a variety of applications to reconstruct volume objects from scattered data and a 3-D space transformation technique. We then consider the so-called *tracking* or *correspondence problem* by applying genetic optimization for automatic correction of corresponding points to avoid superfluous folding over a blended area. The original surface and the restored surface must smoothly join along predefined boundary regions of the restoration area. Genetic optimization was used in the paper to illustrate the approach; nevertheless, other methods such as dynamic programming can be used.

(a) (b) (c)

Figure 2. Tracking problem

To illustrate our approach and the tracking problem, consider the example in Figure 2(a): a black squire is used to "fill" the white area by contracting the white squire in eight directions for gluing black and red areas. Human knowledge and experience is necessary to assign correspondence points to compensate for the gap between the region of restoration and the existing part of the object. Figures 2(b) and 2(c) illustrate the fact that the result of restoration depends on a sufficiently good initial guess. An approach where the target landmarks or destination points are freed to slide along lines discussed by Bookstein in [32], [33] naturally can be used in such a simple case. Figure 2(c) illustrates an application of the weighted least squares technique to select destination points to adopt the configuration of minimum bending energy. In this paper, correction of coordinates of a limited number of corresponding points initially assigned by the user that define the space transformation is done by using a genetic algorithm. One of the most attractive features of genetic algorithms is that they are easy to interface to a model, thus we use genetic optimization as it was discussed in [1]. The book by Vose [34] and the article [35] provide an overview of theoretical aspects of genetic algorithms.

2. 3D-Reconstruction and transformation

In this paper we address the problem of reconstructing missing parts of a treated tooth represented by elevation data on an 2-dimensional grid where the surface is written as the graph of the function: $z - f(x,y) \geq 0$. We assume that due to continuity in the surface the boundary of an existing surface can be propagated to complete the missing part. In order to obtain the proper surface of a treated tooth, a dentist may choose to create and modify the surface in a CAD application interactively using his own experience. Actually, this approach is currently commonplace in dental CAD systems based on parametric representation. A different approach described in [1] is to use the displacement of M control points between model tooth and treated tooth. Authors have shown and proved that they have a good alliance of geometric modeling and optimization techniques to determine the reconstructed surface and assure overall smoothness. Nevertheless, this approach has the defect that a generic tooth acquired from the library of standardized teeth should be used to reconstruct missing part of the treated tooth. This paper is concerned in an algorithm for multi-step extension of an original surface to construct discrete points needed for reconstructing missing part of volume object. After that, for instance finite element approximation [7] for 2D data set or compactly supported RBFs [30] for 3D case can be used. The automatic production of synthetic points reminds an extraction of edges that leads to the "filling" of absent space. Naturally, we do not expect to exactly reconstruct the missing part of an object; our goal is to get outlines that look reasonable. Under the term "reasonable outline" we imply first of all that we wish to extend local features of starting points as globally as possible, and secondly, to avoid superfluous folding over the blended area.

In this section, we shall give a short account of the shape transformation method used in the application considered in this paper. According to our knowledge, the first publication on using discrete landmark points is that of Bookstein [32], see also [33]. To interpolate the overall displacement, we use a volume spline based on Green's function. A detailed description of the method can be found in [10]. The problem is to find an interpolation spline function $U \in W_2^m(\Omega)$, where $W_2^m(\Omega)$ is the space of functions whose derivatives of order $\leq m$ are square-integrable over $\Omega \subset \mathbf{R}^n$, such that the following two conditions are satisfied: (1) $U(p_i) = h_i,\ \ i = 1,2,\ldots,M,$ and (2) U minimizes the bending energy, if the space transformation is seen as an elastic deformation, i.e.,

$$\Phi(U) = \int_\Omega \sum_{|\alpha|=m} m!/\alpha!\ \ (D^\alpha U)^2\, d\Omega \rightarrow \min,$$

where m is a parameter of the variational functional. The minimization of the functional $\Phi(U)$ results in a system of linear algebraic equations for the spline coefficients. An increase in the parameter m essentially leads to an increase in surface rigidity. Our experiments have shown that using the parameter $m = 3$ leads to superfluous folding over the blended area.

A space mapping in $\mathbf{R}^n$ defines a relationship between each pair of points in the original and deformed objects. Let an n-dimensional region $\Omega \subset \mathbf{R}^n$ of an arbitrary configuration be given, and let Ω contains a set of arbitrary control points $\{\boldsymbol{q_i} = (q_1^i, q_2^i, \ldots, q_n^i) : i=1,2, \ldots, M\}$ for a non-deformed object, and $\{\boldsymbol{d_i} = (d_1^i, d_2^i, \ldots, d_n^i) : i=1,2, \ldots, M\}$ for the deformed object. It is assumed that the points $\boldsymbol{q_i}$ and $\boldsymbol{d_i}$ are distinct and given on or near the surface of an object. These points establish a correspondence between features of the two objects under consideration. The goal of the construction of the deformed object is to find a smooth mapping function that approximately describes the spatial transformation. The inverse mapping function that is needed to transform surfaces can be given in the form

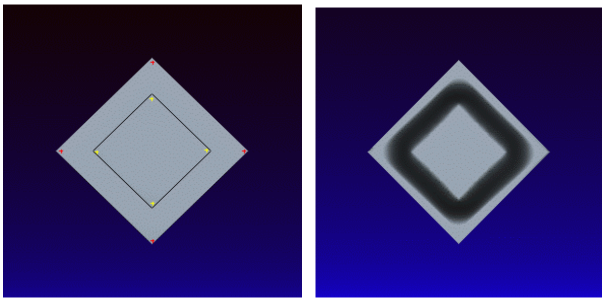
$$\boldsymbol{q_i} = U(\boldsymbol{d_i}) + \boldsymbol{d_i},$$

where the components of the vector $U(\boldsymbol{d_i})$ are volume splines interpolating displacements of initial points $\boldsymbol{q_i}$.

3. Algorithm and results

Nowadays sheet stamping of parts with thin walls is a widespread technology based on finite element models of plastic deformation of sheet metal. To clarify our strategy let us discuss an analogy between our approach and simulation of a sheet metal forming process for a planar square:

- At first, boundaries of the objects and boundary conditions are setting up, see Figure 3(a); in this example red and yellow crosses define the boundaries and provide fixation of the displacement along the plain surfaces.
- Boundaries of surfaces are stretched in opposite directions that can add material in between the boundaries. Area in between of the boundaries is a blended area.
- The material of the sheet is assumed to be rigid plastic and its deformation is described by space transformation technique discussed above.
- The deformation process is regarded as taking place step by step so that the transition from a known state to a new state takes place with small increments in deformations.



(a) (b)

Figure 3. Illustration of stretching a plane squire sheet

The result of applying of two iterations is illustrated in Figure 3(b). Figure 4(a) presents another example - the treated tooth that has the drilled cavity. Besides, the user placed an additional volume object at the central part of treated tooth, that is, volume objects S_1, S_2 are given, thus we can

- Define control-mapping points $\boldsymbol{q}_\mathrm{i}$ on the surfaces of S_1, S_2 objects and

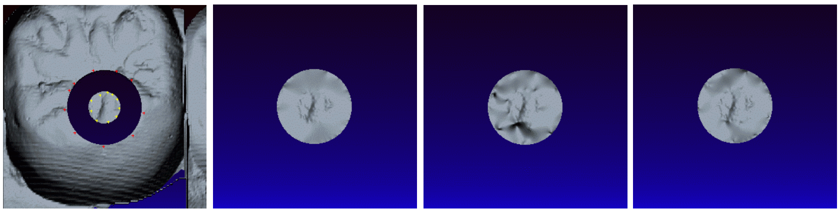
- Define implicit functions for the objects S_1, S_2.

The dark blue area between the additional object S_2 and drill hole is a blended area. The user assigned 20 control points linked to features manually. The small crosses mark these points. Figure 4(d) presents the recovered area of treated tooth. In practice, one can apply several consecutive space mappings, i.e., one can sculpt the desired surface manually, by assigning control points. Unfortunately, smooth surface reconstruction is difficult to realize even for very similar surfaces; the generated surfaces will typically have a sufficiently rough reconstructed surface. To decrease this “roughness”, we apply the following strategy to blend primary surfaces:

- Calculate a mapping function for the current step and apply it to boundary points according to the following procedure:

- Define a point on the object that will be stretched. The point defines the origin of a coordinate system called the master coordinate system, which is used to calculate a stretching direction.
- Features of volume representations are very suitable for calculating gradients. Any vector perpendicular to the surface gradient will define a floating direction, and it is possible to make the starting points float over the surface. The floating or stretching direction is calculated as a bisector direction for two adjacent edges defined by neighboring starting points.
- The plain passing through the starting point and parallel to the vector of a stretching direction, and perpendicular to the floating plane is calculated.
- The line segment is then scaled and x,y,z-steps are calculated. This step actually defines destination coordinates $\boldsymbol{d}_i$ that form boundary contours.

- Apply genetic optimization to avoid superfluous folding over a predefined area.
- For each iteration, calculate new destinations coordinates $\boldsymbol{d}^n{}_i$ and apply the same space mapping technique for points $\boldsymbol{d}^n{}_i$.
- Repeat the loop.



(a) (b) (c) (d)

Figure 4. (a) Tooth to be treated. (b) Area reconstructed by RBFs; All interior boundary and points of central part were processed; (c) Result of reconstruction without optimization; (d) Result of reconstruction with optimization.

The application of the genetic algorithm starts with selecting a set of M variable control points $\{\boldsymbol{d}_i=(d_1{}^i,d_2{}^i,d_3{}^i)$: $i=1,2, \ldots ,M\}$ for the definition of the space transformation generating the reconstructed object. (Actually, for the first iteration the user selects points $\boldsymbol{q}_i$ on the surfaces of the tooth.) The collection of coordinates of $\boldsymbol{d}_i$ defines a creature with 3M coordinates that are subject to correction. The algorithm begins by randomly distorting the initial creature, and generates s creatures, which are collected to form the initial population. Now, the genetic algorithm as described in [1] is applied to this initial setup to minimize the fitness function.

Our main premise here is to avoid superfluous folding over a predefined area where we calculate the fitness to get the "best approximated surface." Recall that the spline (determined by the set of variable control points $\boldsymbol{d}_i$ that constitute a creature) provides a minimization of the bending energy. We define the fitness function f as follows:

$$f(\boldsymbol{c}) = \sum_{p=1}^{M} \sum_{i=1}^{k} \sum_{j=1}^{l} r_{ij} + \boldsymbol{h}^t\boldsymbol{A}^{-1}\boldsymbol{h} \quad \text{for } i,j \in \mathrm{B_f},$$

where the first component of fitness function can be an arbitrarily chosen "gluing" area B_f (here we use small 5×5 pixels image spots), and r_{ij} are residuals (between interpolated and given surface) of elevation teeth data; the second component is the value of the bending energy, where A^{-1} is the bending energy matrix and h_i are so called heights, for the case of space transformations $h_i = q_i - d_i$. The first component of the fitness function expresses the idea that the more smoothly we glue a "sheet" of plastic to a surface, the fewer residuals of elevation data between the blended area and the remained area of treated one will be observed.

Let us note, since we know the new "state" of the surface, the strategy of calculating a stretching direction as a bisector direction for two adjacent edges can be replaced by a more promising approach where we can use geometric information for generating boundary contours.

4. Remarks

In this paper, we have presented an approach for surface reconstruction (more precisely, blending parts of a real geometric object represented by volume data) by employing a space mapping technique based on using RBFs. Figure 4(d) demonstrates the result of reconstruction of the missing part of the tooth according to presented technique. Calculation time is less then one minute on an SGI Octane workstation. We can state that proposed approach allows the design of a blended surface resembling the surface shown in Figure 4(b), reconstructed by using a "naive" RBFs approach. Let us note that processing 2000 points by "naive" RBFs methods takes two hours approximately, while processing of a fulfilled data set with the same number of points by compactly supported RBFs takes less then 10 seconds. There are several directions for future work. Chiefly, we anticipate that modifying the procedure of finding stretching directions by the process of birth and death of destination control points according to the curvature at the point will improve results of reconstruction. It might be a good idea to explore ways in which we will be able to find a good alliance between our approach and a method [36] to design nonlinear splines via curve evolution driven by curvature. While many challenges remain, we believe that the presented approach for recovering missing parts of geometric objects provides an important direction for research and practical applications.

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