

Surface Simplification Based on a Statistical Approach

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Abstract: -This paper presents work in progress and continues a project devoted to developing shape modeling system for surface generation and enhancement. A local mesh enhancement based on statistical characteristics of an initial triangle mesh is proposed for mesh simplification. Experimental results are included to demonstrate the functionality of our simplification algorithm.

Key-Words: - Mesh statistics, simplification, radial bases functions, aspect ratio

1 Introduction

Surface remeshing has become very important today for computer-aided design (CAD) and computer graphics (CG). This question is also very important for technologies related to engineering applications. Simplification of a geometric mesh involves constructing a mesh element which is optimized to improve the element's shape quality.

The final goal of our project is to allow mesh to be generated from scattered point sets for finite element analysis. Various automatic mesh generation tools are widely used for finite element analysis. However, all of these tools may create distorted or ill-shaped elements, which can lead to inaccurate and unstable approximation. Thus, improvement of mesh quality is an almost obligatory step for preprocessing of mesh data in finite element analysis. In spite of a flurry of activity in the fields of mesh modification, this matter remains a difficult and computationally expensive problem.

This paper presents work in progress, and continues a project devoted to developing shape modeling system for surface generation and enhancement. A local mesh enhancement based on statistical characteristics of an initial triangle mesh is proposed for mesh simplification.

We present a surface simplification method which uses predictor-corrector steps for predicting candidate collapsing points with consequent

correcting them by the use of a statistical approach for triangles enhancement. The predictor step is based on the idea of obtaining 3D coordinates according to a bending energy.

We show that since the simplification method is sufficiently efficient for up to 90 percent of reduction, there is no need for user-tuned parameters. We present an approach for obtaining a realistic time response (few seconds on AMD Athlon 1000 MHz) for sufficiently complex models (70K triangles).

The rest of the paper is organized as follows. The next section gives an overview of papers related to this work. We discuss the simplification algorithm in Sec. 3. and Sec. 4, examples are given in Sec 5, and Sec. 6 contains concluding remarks.

2 Related Works

Complex and detailed models can be generated by 3D scanners, and such models have found a wide range of applications in CG and CAD, particularly in reverse engineering. Nevertheless, it is useful to have various simpler versions of original complex models according to the requirements of applications. Recently, a tremendous number of very sophisticated algorithms have been invented to obtain a simplified model. One exceedingly good overview [1] presents

a problem statement and a survey of polygonal simplification methods and approaches. Most existing simplification algorithms use such topological operations as vertex decimation, edge decimation, and triangle decimation. Vertex decimation methods remove a vertex according to a decimation cost defined by an error metric. Examples of the latter include the curvature of a local surface around a given vertex [2]; the Hausdorff distance between the simplified and the original mesh [3]; the distance between a given vertex and the average plane defined by its surrounding vertices [4]. To fill the resulting hole, a robust re-triangulation method is needed. An iterative edge contraction method is usually used in edge decimation methods. These methods use various metrics such as the sum of the squared distance functions between the original mesh and the simplified mesh [5, 6]; the quadric error metric between a vertex and an associated plane as proposed in [7]. Actually, in the edge contraction method, a new vertex can be created at the midpoint of a line between two vertices; an optimal point or an endpoint can also be used to place a new vertex. In [8] an extremely fast simplification method based on a probabilistic optimization technique was proposed.

3 Simplification Algorithm

Metrics based on object geometry properties are mainly used in simplification algorithms. Current simplification algorithms might be made more effective if an algorithm could produce an approximation of the simplified surface. In fact, we present an attempt to combine a simplification process with a surface approximation based on the use of statistics.

Here, we shall give a short account of a space transformation method used in the applications considered in this paper (for further references, see [9]). To interpolate the overall displacement, we use a volume spline based on Green's function. This is the well-known problem of finding an interpolation spline function $u \in W_2^m(W)$, where $W_2^m(W)$ is the space of functions whose derivatives of order $\leq m$ are square-integrable over $W \in \mathbb{R}^n$, such that the following two conditions are satisfied: (1) $u(p_i) = h_i$, $i = 1, 2, \dots, N$, and (2) u minimizes the bending energy, if the space transformation is seen as an elastic deformation.

For an arbitrary three-dimensional area W , the solution of the problem is well-known: the volume spline $f(P)$ having values h_i at N points P_i is the function

$$f(P) = \sum_{j=1}^N \mathbf{g}_j \mathbf{f}(|P - P_j|) + p(P), \quad (1)$$

where $p = \mathbf{n}_0 + \mathbf{n}_1x + \mathbf{n}_2y + \mathbf{n}_3z$ is a degree one polynomial. To solve for the weights \mathbf{g}_j we have to satisfy the constraints h_i by substituting the right part of equation (1), which gives

$$h_i = \sum_{j=1}^N \mathbf{g}_j \mathbf{f}(|P_i - P_j|) + p(P_i). \quad (2)$$

Solving for the weights \mathbf{g}_j and $\mathbf{n}_0, \mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ it follows that in the most common case there is a doubly bordered matrix \mathbf{T} , which consists of three blocks, square sub-matrices \mathbf{A} and \mathbf{D} of size $N \times N$ and 4×4 respectively, and \mathbf{B} , which is not necessarily square and has the size $N \times 4$.

Finding the optimal decimation sequence is a complex problem. The traditional strategy is to find a solution that is close to optimal; this is a greedy strategy, which involves finding the best choice among all candidates. Our simplification algorithm is sufficiently simple and is based on an iterative procedure for performing simplification operations. In each iteration step, candidate points for an edge collapse are defined according to a local decimation cost of points belonging to a shaped polygon. We call such a polygon a star. After all candidates have been selected, we produce a contraction step by choosing an optimal point.

In our implementation, we apply a specific error metric. We propose using the bending energy $\mathbf{h}^T \mathbf{A}^{-1} \mathbf{h}$ as an error/quality cost to select candidates for an edge collapse.

We employed an approach that uses displacements of N control points as the difference between the initial and final geometric forms. The central point of a star polygon is considered as a point that can slide to the neighboring points. The selection of candidate points is made according to the bending energy. We exploit a simple idea that the more smoothly we transform a central point, the fewer residuals there will be between an initial mesh and the subsequent mesh. In this step, we form a list of points to be contracted; this list contains a number of candidate points. In the contraction step we eliminate processing of points that can be contracted twice or more.

Vertex placement is produced in two steps. In the first step we generate a position on the line connecting two vertices of an edge to be contracted. In fact, the optimal point is generated at the next step according to the following scheme:

- Define a tangent or an average plane passing through the point calculated in the previous step. Once we have information about the neighboring points, which can be achieved by estimating the tangent plane, we can estimate the local surface properties. Hoppe et al. [10] have demonstrated that eigenvalues $\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3$ of the covariance matrix of neighboring points could be used to produce normal estimation. The eigenvalues $\mathbf{I}_1, \mathbf{I}_2$, and \mathbf{I}_3 define also

an orientation of an ellipsoid containing the data. The values I_1 , I_2 , and I_3 describe the surface variation \mathbf{s} and can serve as an analog of a surface curvature that is used for generation of polygonal data sets to make a decision about the complexity of the local reconstruction area. Assuming that I_1 is minimal, I_1 describes the variation along the surface normal, and the directions corresponding to the eigenvalues I_2 , I_3 define a tangent plane; we proceed as follows:

- Apply a rotation to the neighboring points so that the nearest plane is perpendicular to the z -direction.
- Calculate a new vertex position (see Section 4).
- Apply a transformation for the selected point.
- Produce an inverse rotation of the calculated point.

Let us note that the surface variation \mathbf{s} can be used as an estimation of a local curvature of current mesh to distinguish the areas having different geometric characteristics for preserving object features. Afterwards, when the original model has been simplified, we repeat the above steps to produce new updates.

4 Corrections of Predicted Points

The main underlying assumption of the algorithm is that a local mesh refinement automatically results in improved global mesh quality, bearing in mind the distribution of a limited set of polygons in the entire model.

We use statistics on the values of the mesh quality criteria parameters of the neighbors of each vertex of the triangle mesh in order to predict the most likely value. This provides some latitude in the choice of point placement allowing softer “transformations” of polygons to be produced. In each simplification step, a point belonging to a shaped polygon formed by union of two polygonal shapes (see Fig. 1) is processed in accordance with a “statistics” mapping function. The shaped polygon is created by union of two stars. To calculate the “statistic” mapping function we rotate the polygon area points so that the tangent plane for the area is perpendicular to the z -axis.

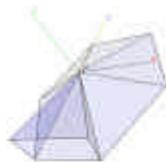


Fig. 1. Illustration of an oriented pyramid.

Statistics deal with real numbers which correspond to individual elements of the polygonal mesh. These numbers are recognized in the algorithm as values of mesh quality criteria

parameters. In our current work the mesh quality criteria parameter is calculated in 2D space, where 3D coordinates of local vertices are projected onto local average plane and defined as a ratio of the maximum edge length to the minimum edge length of an element of the triangle mesh (m/mc). This definition ensures a range of m/mc values from 1 to the maximum value. This range, denoted here as $X = [1, \max]$, is called probabilistic space. Usually, in statistics with a large amount of random sampling, experimental data are grouped into special intervals showing the number of elements that lie in such intervals of probabilistic space fragmentation. Since the empirical distribution of experimental data is discrete, we consider the frequency histogram as an analogue of density function over this space x . Considering the initial distribution of m/mc values, we assume that after refinement the distribution varies from a rather random distribution to a smoother one, such as a Gaussian normal distribution. A special mapping of an empirical distribution function to the desired smooth distribution (see Fig. 2) transforms m/mc values from the initial frequency histogram to a potential distribution (actually, one can require various distribution functions). This choice of distribution affects the new m/mc values predicted by probabilistic formulation.

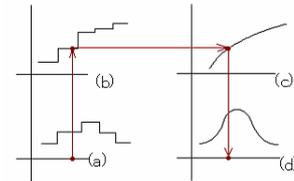


Fig. 2. Mapping of a distribution function: (a) initial density; (b) initial distribution ; (c) predicted distribution; (d) predicted density.

A technique is applied for an oriented pyramid: a star polygon, as it shown in Fig. 3, centered at a processed point. The new center of a star with respect to its neighbors is calculated for each star by statistical means. We can consider a triangle element in the star with a fixed boundary and let the inner point of the star freely slide. Afterwards, for each element we find a new center that provides a potential shape of the element. This means that the element can have m/mc value found by prediction. These values are the ideal ones with respect to the star’s neighbors. Finally, new center coordinates are the average values of potential center coordinates. Statistical parameters and characteristics of the global mesh quality shown below demonstrate the applicability of the proposed method. In the algorithm described, for each star with n elements and its m/mc values equal a_1, \dots, a_n one can apply probabilistic analysis over intervals $[n_j, n_{j+1}]$

(for some integer number k , where $n_0 = a_{\min}$, $n_k = a_{\max}$) to calculate the distribution (histogram) in a current star and calculate new values a_1^*, \dots, a_n^* predicted by formula (3) (see [11])

$$a_i^* = \sqrt{-2D \log(1 - F_j)} + 1, \quad (3)$$

where D is the variance or deviation.

Statistical parameters such as the deviation D of m/mc values from the average value M in the star, a probability F_j for the m/mc values a_i lying in interval $[n_0, n_{j+1}]$ can be found as follows:

$$M = \frac{\sum_{j=1}^n a_j}{n}, \quad D = \frac{\sum_{j=1}^n (a_j - M)^2}{n}, \quad (4)$$

$$f_j = \{i : n_j < a_i \leq n_{j+1}\}, \quad (5)$$

$$F_j = \frac{\sum_{i=1}^j f_i}{n+1}, \quad (6)$$

where f_j is a histogram value.

Fig. 3 shows, from left to right, an initial star, potential triangles in the star, and averaged coordinates of inner potential points for an arbitrarily chosen star.

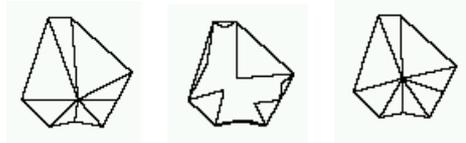


Fig. 3. An eight-triangle star.

After processing an entire fragment of the model in accordance with the procedure described above, histogram values f_j are calculated for the fragment of a model. Fig. 4 shows three examples that we investigated: an initial fragment, the improved fragment obtained by averaging according to equilateral triangles ($a_i^* = 1$) in each star, and the fragment after processing. From the histograms below we can see that although the average value M is better in the case (b) than in the case (c), the

distribution of m/mc values approximates a Gaussian normal distribution. This means that the algorithm allows us to realize a gentle mesh improvement and to provide a rather smooth correction of the histogram.

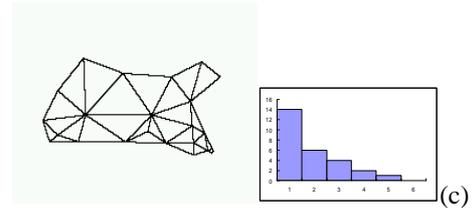
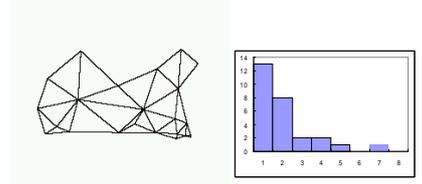
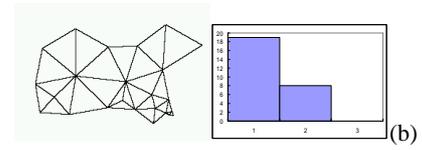
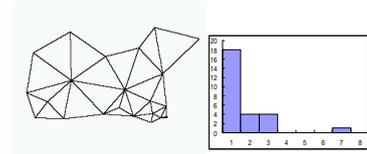
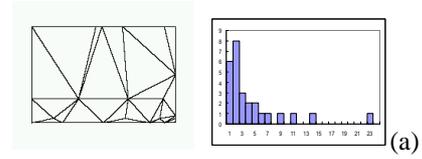


Fig. 4. Mesh fragments and frequency histograms: (a) Initial fragment ($M = 2.84$, $D = 5.76$), (b) According to $a_i^* = 1$ (one step, $M = 1.45$, $D = 0.34$; two steps, $M = 1.34$, $D = 0.06$), (c) Use of statistics (one step, $M = 1.78$, $D = 0.51$; two steps, $M = 1.68$, $D = 0.37$).

5 Examples

We demonstrate our work on two models. Fig. 5 shows examples of polygon simplification of a technical detail.

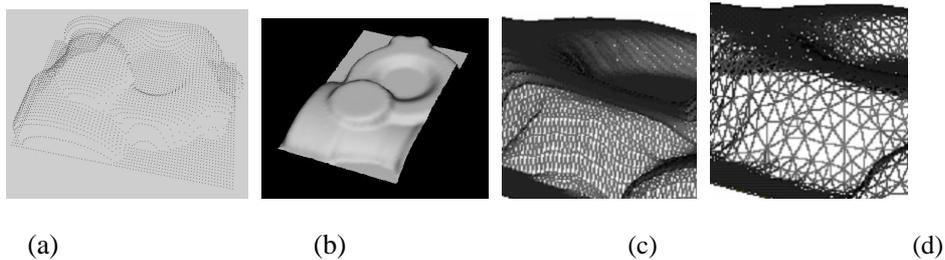


Fig. 5. Surface of a technical data set: (a) Cloud of points (4100 scattered points are used);

(b) Simplified mesh shaded; (c) Fragment of the initial mesh, 31234 triangles;
 (d) Combined mesh modification (polygon reduction and statistical improvement of the mesh), 12132 triangles.

Fig. 6 and Fig. 7 demonstrate that rather homogenous distribution of triangles can be attained.

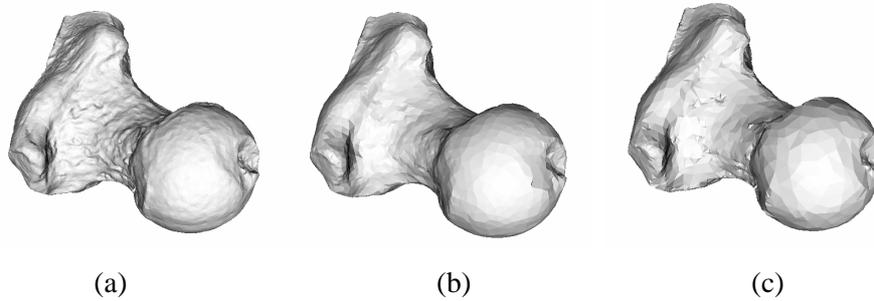


Fig. 6. Surface of the “ballJoint-25” model (Cyberware Inc.) : (a) Initial mesh. Number of triangles 68530; (b) After simplification. Number of triangles 7320; (c) After simplification with preserving of sharp features.

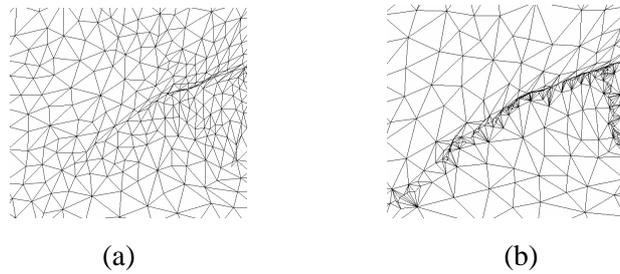


Fig. 7. Wire-frame fragments: (a) Simplified mesh of the “ballJoint-25” model (see Fig. 6 (b)); (b) After simplification with preserving of sharp features.

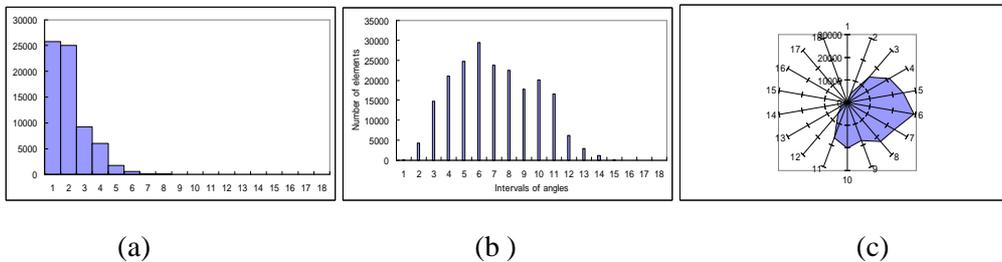


Fig. 8. Initial mesh: (a) Frequency histogram of the aspect ratio ($M = 1.75$, min aspect ratio = 1.00, max aspect ratio = 5.96); (b) Frequency histogram of the element angles (intervals from 3 to 14 contain the angle values over the range 30 to 120 degrees; min angle = 3.22 degrees, max angle = 158.51 degrees); (c) Radar type histogram of the angles.

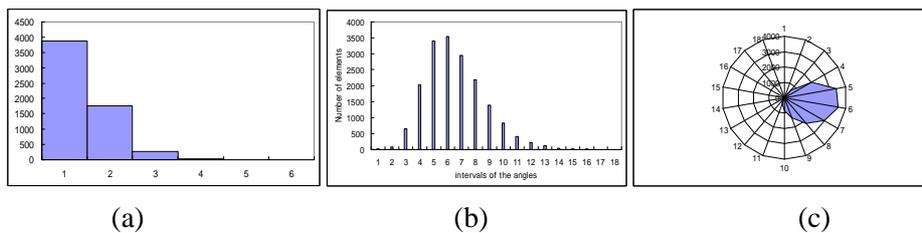


Fig. 9. Mesh after simplification: (a) Frequency histogram of the aspect ratio ($M=1.38$, min aspect ratio = 1.00, max aspect ratio = 3.65); (b) Frequency histogram of the element angles (intervals from 3 to 12 contain the angle values over the range 30 to 120 degrees; min angle = 5.57 degrees, max angle = 167.88 degrees); (c) Radar type histogram of the angles.

Fig. 8 and Fig. 9 demonstrate mesh quality parameters and statistics of the “ballJoint-25” model.

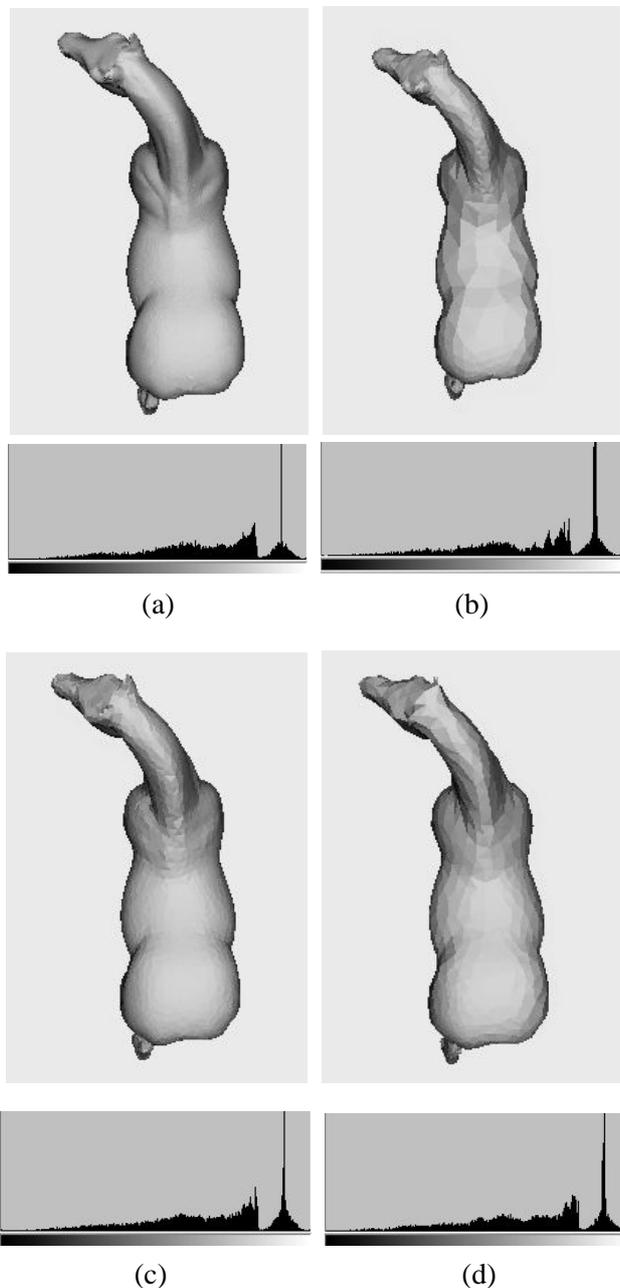


Fig. 10. (a) Original “horse” model (96966 triangles) and a luminosity histogram, (b) Simplified model produced accordingly to the use of the bending energy (10% of the original number of triangles) and the luminosity histogram, (c) Simplified model produced accordingly to the use of the bending energy and statistical approach (9% of the original number of triangles) and the luminosity histogram, (d) Simplified model produced accordingly to the use of the bending energy and statistical approach (3% of the original number of triangles) and the luminosity histogram,

6 Conclusion

There is no single simplification method that provides the best results for every surface in the sense of quality and processing time. Moreover, the question of whether the mesh is appropriate for FEM is an open question. It was mentioned in [12] that “...forty-odd years after the invention of the finite element method, our understanding of the relationship between mesh geometry, numerical accuracy, and stiffness matrix conditioning remains incomplete, even in the simplest cases.”

In presented paper, we have considered only geometric solution to attain a more uniform (homogeneous) mesh in the sense of histogram distribution of elements calculations.

Experimental results indicate that the algorithm proposed in this paper provide rather good results and look promising for implementation in CAD and computer-aided engineering applications. It is proposed to use the bending energy $h'A^{-1}h$ as an error/quality cost to select candidates for edge collapse and produce a final vertex placement according to statistics approach. Let us note here that, for example, in Fig. 5, the volume was rather well preserved, with a difference between the initial mesh and the processed one of about 0.65%. We compare our approach with the software algorithm from [13]. Our simplification algorithm does not provide such a good quality of simplification as the algorithm given in [13]. However, it has the advantages that the processing time is almost three times as fast, and no user-tuned parameters are required. At the same time, it guarantees overall smoothness, exhibits a good visual appearance that is verified the luminosity histogramm (see Fig. 10(c)), and preservation of features. Experiments show that features of an objects can be preserved in simplification steps while statistics approach is used.

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